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INSTRUCTION MANUAL

FOURIER ANALYSIS KIT™
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FOURIER ANALYSIS KIT

INTRODUCTION

In electronics we very often come across signals, which are not simple sine waves. The waveforms of such signals are complex. They contain a number of harmonics. To process such signals it is necessary to know their frequency components and their relative magnitudes. The mathematical tool that does this type of analysis is the FOURIER THEOREM.

According to the Fourier theorem, any single valued complex periodic waveform ($V_{(t)}$) can be thought of as a sum of a series of simple harmonic waves, the first of which has a frequency equal to that of the complex wave.

$$V_{(t)} = V_{dc} + \sin(\omega t + \phi_2) + \ldots + V_n \sin(\eta\omega t + \phi_n). \quad \ldots(1)$$

The term V_{dc} is called the dc level of the signal. The terms V_1, V_2. V_n are the amplitudes of the fundamental or the first harmonic (ω), the second harmonic (2ω) and so on. The terms $\phi_1, \phi_2 \ldots \phi_n$ are the corresponding phase angles.

The Fourier series of a square wave is given by

$$V(t) = \frac{4V}{\pi}(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \frac{1}{7}\sin 7\ \omega t + \frac{1}{9}\sin 9\omega t + \ldots) \quad \ldots(2)$$

This equation shows that the square wave is made up of sine waves of the fundamental and its odd harmonics. If the amplitude of the fundamental is taken as unity, the amplitudes of the third harmonics is 1/3, that of the fifth harmonic 1/5 and so on.

The Fourier series of a triangular wave is given by

$$V(t) = \frac{4V}{\pi}(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \cdots) \quad \ldots..(3)$$

The triangular wave contains the fundamental and its odd harmonics. The amplitude decreases as $1/n^2$ where n is the number of the harmonic (odd).

The Fourier series of a clipped sine wave (half-wave rectified) is given by

$$V(t) = \cos\omega t - 0.425\cos 2\omega t - 0.085\cos 4\omega t - 0.03\cos 6\omega t\ldots.. \quad \ldots(4)$$

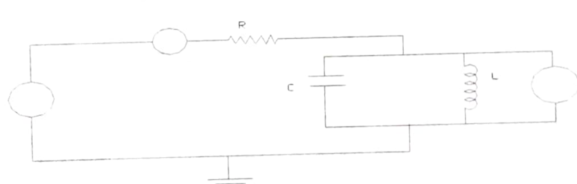
The theoretical relative amplitudes of the Fourier components of the square, triangular and clipped sine wave forms are given in table 1.

TABLE 1. **FOURIER COMPONENTS**

Harmonic	Amplitude (Relative)		
	Square Waves	Triangular waves	Clipped sine waves
First	1.00	1.00	1.00
Second	---	---	0.43
Third	0.33	0.11	---
Fourth	---	---	0.09
Fifth	0.20	0.04	---
Sixth	--	---	0.04
Seventh	0.14	0.02	---

METHOD OF ANALYSIS

The circuit used to analyse complex waveforms is shown in Fig. 1. The waveform to be analyzed is given to the series combination of a resistor and tuned circuit whose resonant frequency (ω) $\simeq$ 7.5 kHz. The tuned circuit is specially designed to achieve high selectivity at this frequency. It selects this frequency and rejects all other frequencies with high efficiency. The voltage across the tuned circuit is given to the vertical input of a CRO.



When the frequency of the input waveform is equal to the resonant frequency of the tuned circuit, the circuit accepts the fundamental component and rejects all other harmonics. As a consequence, a pure sine wave of that frequency is seen on the CRO. As the frequency of the input wave decreased, a pure sine wave of frequency is will appear on the CRO whenever $\omega' = \frac{\omega}{n}$ where $n = 2, 3, 4, \ldots$. The integer n takes only odd values for the square and triangular waveforms. It takes only even values for the clipped sine wave.

A measurement of the frequency of the harmonic and its relative amplitude enables one find out the component harmonics.

THE KIT

The kit consists of a stabilized dual power supply unit, a function generator and the analyser. The power supply system, the function generator and the analyser are housed in main unit.

Square, triangular and sine waveforms are generated by a specially designed integrated circuit function generator whose output frequency can be varied in the range of 500 Hz to 15 kHz. The output level of the sine and triangular waves can be varied using the amplitude potentiometer provided on the front panel of the main unit.

The frequency of the generator can be varied using the 100 kΩ ten-turn potentiometer (TTP). Use of the TTP achieves smooth frequency variation.

The frequency of the generator for any particular setting of the TTP is given by

$$F\,(\text{kHz}) = \frac{44.4}{1+R} \qquad \ldots(5)$$

where (1 + R) is in kΩ. R is the resistance of the TTP. If the dial of the TTP reads 1.02, its resistance is 10.2 kΩ and (1 + R) is equal to 11.2 kΩ leading to a value of 4.46 kHz for the frequency of the waveform. Equation (5) yields frequency values with an accuracy of $\pm$ 5%.

FRONT PANEL

The front panel of the main unit containing the function generator and the analyser circuit is shown in Fig. 2.

The output waveforms of the function generator are available at sockets A, C & D. The square wave is available at A. At C, the waveform will be either a sine wave or a triangular wave depending upon the position of the toggle switch S. The frequency of all the three waveforms may be varied using the TTP (ten turn potentiometer).

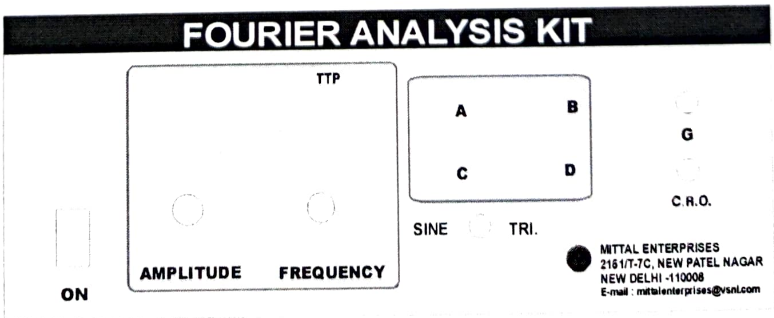


Fig.2

B is the input to the analyser circuit. The sockets marked G is ground terminals. The output of the analyser is available at socket marked CRO.

EXPERIMENTAL PROCEDURE

1. Connect the unit to the mains.
2. Connect CRO and to the Unit.
3. Waveform of the function generator can be seen between the respective terminals and G.

SQUARE WAVE

4. Connect socket marked A to B using the patch cord supplied with the kit, this feeds the analyser with the square wave.
5. Observing the pattern on CRO vary the TTP starting from the zero reading. This varies the frequency of the square wave. At *f*, the natural frequency of the tuned circuit, you will observe a pure sine wave of maximum amplitude. Adjust TTP carefully to get maximum amplitude. Note the reading of the TTP and measure the height of the pattern on the CRO. The height of the pattern may be measured in volts if your CRO has a valibrated scale. Otherwise measure the height using a scale or divider. The height is proportional to the amplitude of the pure sine wave.
6. Decrease the frequency of the waveform and note the reading of the TTP for which you observe pure sine waves of maximum amplitude on CRO. Measure their amplitudes you would observe pure sine wave at 1/3, 1/5, 1/7 and 1/9 and so on of ω .

   The result shows that a square wave contains the fundamental and its odd harmonics. Tabulate your results as indicated in Table 2.

TABLE 2. **WAVEFORM: SQUARE**

TTP Reading	R (kΩ)	(1+R) (kΩ)	Frequency (kHz)	Harmonic	Amplitude

Disconnect A and B

TRIANGULAR WAVE

7. Throw the toggle switch towards the sign of triangle. Adjust the amplitude potentiometer so as to get undistorted triangular waves. Connect the CRO to terminal C and observe the waveform . You will now get triangular wave at the socket marked C. Remove the CRO connection from C and connect it to the terminal marked CRO.

8. Connect C to B using a patch cord.

9. Repeat steps 4 and 5 described above.

CLIPPED SINE WAVE

10. Throw the toggle switch towards sine sign. You will now get sine wave output at C. There is a half-wave rectifier between C and B. At D we get the clipped sine wave.

11. Connect D and B. The analyser is now fed by the clipped sine wave.

12. Repeat steps 4 and 5 described above.

PRECAUTIONS

1. The frequency of the waveform should be adjusted carefully to obtain maximum amplitude. This is particularly important at the fundamental. Any mistake here leads to errors in the amplitudes of other harmonics.

2. At higher harmonics the amplitude decreases to low values. For some harmonics the error in measurement may be comparable with the amplitude that is being measured. So no importance need be attached to exact agreement between the theoretical and experimental relative amplitudes. We should only look for their existence.

3. Any distortion of the input waveform leads to erroneous results. If there is a large discrepancy between the experimental and theoretical results examine the waveforms using the CRO. If you find serious distortion of the waveform, inform the company. Arrangements will be made to correct the waveforms.

4. Do not use the function generator for other experiments. It may overload the generator and distort the waveforms.

SUGGESTION

You may use the analyser to analyse the waveforms from any other source. Feed the waveforms between B and G.

TYPICAL RESULTS

Square wave

TTP Reading	R in kΩ	(1+R) in kΩ	f (kHz)	Harmonic	Amplitude
0.49	4.9	5.9	7.52	I	8.80
1.68	16.8	17.8	2.49	III	2.85
2.87	28.7	29.7	1.49	V	1.75
4.05	40.5	41.5	1.07	VII	1.20
5.25	52.5	53.5	0.83	IX	0.99

Triangular Wave

TTP Reading	R in kΩ	(1 +R) in kΩ	f (kHz)	Harmonic	Amplitude
0.49	4.9	5.9	7.52	I	3.20
1.67	16.7	17.7	2.50	III	0.33
2.86	28.6	29.6	1.50	V	0.125

Clipped Sine Wave

TTP Reading	R in kΩ	(1 +R) in kΩ	f (kHz)	Harmonic	Amplitude
0.49	4.9	5.9	7.52	I	0.66
1.07	10.7	11.7	3.79	II	0.25
2.26	22.6	23.6	1.88	IV	0.058

References

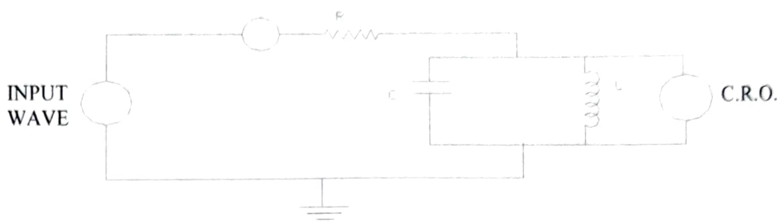
1. Mathematics of physics and modern Engineering by I.S. Sokolnikoff and R.M. Redheffer.

2. "Experiments" in Electronics by S.V. Subrahmanyam, Mac Millan, India, 1983.

A NOTE ON THE PROCEDURE ADOPTED TO STUDY FOURIER COMPONENTS OF A COMPLEX WAVE

1. A straightforward method of studying the Fourier Components of a complex wave like the square wave is described below:

   The experiment setup is shown below:



   In this set up, a square wave of fixed frequency feeds the series combination of a tuned circuit consisting of an inductance L, a variable capacitance C and a resistance of R. The voltage developed across the tuned circuitry is displayed on C.R.O.

2. A square wave consists of pure sine wave of fundamental f, third harmonic $3f$, fifth harmonic $5f$, seventh harmonic $7f$ and so on. Their amplitude decreases in the order 1/3, 1/5,1/7 etc.

3. If we now tune the tuned circuit to f we will observe a pure sine wave of frequency f, its amplitude depends on Q-factor of the circuit, i.e. on $\frac{8 \cdot \pi \cdot f \cdot L}{R}$.

   If L & R remains constant, the fundamental will be unity.

4. Let us now tune the circuit of $3f$, then a pure sine wave of frequency $3f$ will be observed. It indicates the existence of the 3rd harmonic. However its amplitude will not be 1/3 of its fundamental due to the fact that the Q of the circuit is 3 times the Q of the fundamental.
   Same thing happens when the circuit was tuned for $5f$. 5th harmonic will be observed but its amplitude will not be 1/5 of its fundamental.

Thus the procedure described above gives a wrong idea of the relative amplitudes of the Fourier components even though it shows the existence of the higher harmonics in a straightforward manner.

To avoid the above difficulty we have adopted a procedure that is not so straight forward but clearly and accurately shows the existence of the harmonics and their relative amplitudes. In the KIT, we have kept the resonant frequency of the tuned circuit at fixed value of f, and input frequency of the signal is varied. Since we are not changing the resonant frequency of tuned circuit its Q remains constant.

5. While varying the input frequency, when the frequency of the input square wave signal is f, a sine wave of frequency f will be observed o n C.R.O. Let its amplitude be unity.

6. If now the input frequency is decreased to $f/3$, a pure sine wave of frequency f will again be observed on C.R.O. This indicates the existence of the third harmonic $\frac{1}{3f} \times 3f = f$. Since the frequency of the wave appearing across the tuned circuit is again f, its Q does not change. Hence its amplitude will be $1/3^{rd}$ of the amplitude of the fundamental, as the theory predicts.

7. If we now decrease the frequency to $f/5$, again a pure sine wave of frequency f will be observed i.e. 5^{th} harmonic ($\frac{1}{5f} \times 5f = f$) and so on.

The above explanation justifies the procedure adopted in the KIT for Fourier analysis of complex waves.